THE ASSOCIATION OF VOLUNTARY MOVEMENTS

A Dissertation Submitted in Partial Fulfillment of
the Requirements for the Degree of
Doctor of Philosophy

BY

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A CLASSIFICATION OF GROUPS[1]

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INTRODUCTION

The present study is based upon the work of Swindle[2] who described and defined certain very simple and characteristic movement complexes to which he gave the name *groups*. As a provisional definition of a *group* he says as follows: "The simplest conceivable instinctive movement (Bewegungsinstinkt) is the result of the capability of an organism to react so many times with a given member of the body (Koerperglied) until a definite number of movements have been made"—that is to say, until a *group* of similar movements have been beat in a particular tempo, amplitude, and direction.

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The writer wishes to take the opportunity of expressing his obligations to his observers, namely, his wife, Dr. Karl Dallenbach of Cornell University, Dr. Geo. F. Arps of Ohio State University, and to Miss Ruth Miller, graduate student of the latter institution; to Dr. A. P. Weiss who read the manuscript and whose constant help and guidance made this work possible; more particularly, however, to Dr. G. F. Arps whose liberality of thought and action and constant interest and attention may not be overestimated.

[2] Swindle, P. F. Zeitschrift für Psychologie u. Physiologie der Sinnesorgane, 1915, Bd. 49.

In amplification of the above, Swindle's observations on certain animals may well be cited. He observed that certain owls (Glaucidium whitelei Lws.) made characteristic periodic to and fro movements with the tail. More concretely stated, the owls moved the tail to and fro a certain number of times and then stopped; after a pause of variable duration they made similar movements and rested, and so on throughout longer periods of observation. Other animals behaved similarly, some using this, others that member of the body, or limb, for the performance of these group movements.

By definition any series of periodic movements of the same body members is called a *group*, and any single to and fro movement, a *beat* or an *element* of a group. Moreover, if the group consists of 5 elements or 8 elements, or 23 elements, etc., it is respectively known as the 5-group, 10-group, 23-group, etc., to distinguish it from groups which contain any other number of elements.

The term *group* had however another justification than that of merely defining or describing a series of similar periodic movements. It had a functional or behavioral justification which was based upon the fact that certain groups recurred very frequently in the same animal, and indeed far more frequently than would be expected, were the frequency of recurrence only a matter of chance. In consequence of the apparent stability of these activities the term *group* carries with it the implication that groups are entities of some kind,—functional units of perhaps the same order as the more traditional functional units commonly called reflexes, instincts, and habits. Indeed, Swindle identifies groups with instincts and habits accordingly as they are inborn or acquired.

The observers whose data constitute the basis of the present study were human beings. They were instructed by the writer to beat periodically on the button of a tambour as long as they desired to do so: to rest as long as they had previously beat: and finally, so to alternate beating and resting until instructed to cease. No limitations were placed on the periodicity of their

beating nor upon the duration except that the periodicity of any given group was to remain constant.[3]

This procedure gave as its data (1) an alternate series of groups and rests, and (2) a series of discrete but contiguous durations, i.e., durations of groups, and durations of rest periods. Generalized the data would appear as follows:

(1) Group, Rest, Group, Rest, G, R, G, R, G, R, etc.
(2) G–Dur., R–Dur., G–Dur., R–Dur., G–D, R–D, G–D, R–D, etc.

Quantitatively expressed, the above two series may be expressed:

(3) a–G, R, b–G, R, c–G, R,..........w–G,R, x–G, R, y–G, R, z–G
(4) a–sec., b–sec., c–sec., d–sec., e–sec., f–sec., g–sec., etc.

where *a*, *b*, *c*, *d*, etc., in (3) stand for simple numbers, each describing and defining the group to which it has reference from the point of view of the number of elements contained in it; and where the corresponding coefficients in (4) qualify the durations of both groups and rests in seconds. Inasmuch as the rest periods are not otherwise quantitatively described in series (1) and (3), they may be entirely disregarded in these equations or series, and the latter may be written as follows:

(5) a–G, b–G, c–G, d–G,............w–G, x–G, y–G, z–G

The above series only describes groups as to the number of elements or beats contained in them; it says nothing regarding the other characteristic properties of groups, such as their periodicities, or their durations; it disregards entirely the rests and their durations, but it is the fundamental series of this paper, which seeks to answer the following questions:

(a) What groups are beat by any observer?
(b) Is a general classification possible?

Our observers were seated in one of the standard tablet chairs, the right fore-arm resting on the tablet, the index finger crooked

[3] It goes without saying that observers were not permitted to count either silently or aloud while beating.

and resting upon the button of a *cardio-tambour*. The said tambour was fastened to the extreme front of the tablet in a convenient place for tapping, and was connected to a more complicated recording tambour by a rubber tube, as ordinarily in such cases. Records of the excursions of the pointer of the recording tambour were made upon smoked paper which was stretched between two horizontal drums. One of the latter

TABLE 1

Observer R

1.) 44, 24, 44, 47, 22, 46, 52, 48, 68, 32, 104, 47, 7, 45, 10.
2.) 36, 54, 27, 36, 96, 115, 174, 130, 49, 12, 17, 16, 24, 52, 45, 21, 18, 130.
3.) 23, 28, 42, 45, 46, 46, 46, 22, —, 19, 44, 68, 44, 44, 70, 143, 190, 10, 10.
4.) 22, 115, 22, 22, 45, 24, 32, 8, 93, 156, 106, 99, 47, 31, 79.
5.) 77, 46, 22, 76, 393, 18, 22, 22, 46, 156, 7, 7, 7, 19, 22, 46, 68, 46, 46, 8.
6.) 20, 43, 45, 22, 94, 6, 7, 7, 7, 7, 6, 7, 50, 3, 3, 3, 3, 45, 45, 22, 46, 46, 46, 46, 46.
7.) 44, 10, 49, 68, 50, 48, 48, 46, 46, 46, 46, 68, 71, 22, 22, 46, 46.
8.) 22, 75, 70, 46, 46, 70, 115, 96, 108, 71, 71, 22, 46, 46, 46, 46, 46, 46, 46, 46.
9.) 46, 70, 46, 117, 22, 70, 70, 333, 22, 22, 22, 22, 22, 70.
10.) 29, 31, 31, 15, 15, 22, 22, 22, 21, 22, 47, 46, 70, 22, 22, 22, 46, —, —, 75, 22, 47, 60
11.) 46, 46, 47, 45, 46, 8, 8, 46, 22, 22, 22, 22, 22, 22, 22, 22, 96, 94, 46, 98, 47, 46, 46.
12.) 48, 136, 118, 94, 100, 47, 174, 94, 70, 46, 46, 94, 94.
13.) 22, 22, 22, 23, 22, 46, 46, 46, 24, 22, 46, 45, 142, 146, 95, 60, 66, 46, 70, 95.
14.) 46, 94, 94, 68, 95, 46, 46, 47, 416, 48, 94.
15.) 22, 22, 46, 94, 46, 68, 94, 94, 94, 48, 46, 46, 49, 94, 70, 32, 44.
16.) 43, 3, 3, 3, 3, 3, 3, 3, 3, 13, 12, 12, 10, 10, 14, 20, 22, 22, 44, 74, 46, 46.
17.) 47, 47, 46, 133, 94, 79, 94, 118, 95, 94, 70, 46, 46.

formed part of an electrically driven kymograph, the other was simple and revolved upon a fixed axis which was supported on two stands by clamps. The two drums were placed on separate tables which were some 15 to 20 feet apart. The recording tambour as also the time signal were fixed in front of one of the drums to a platform movable in the direction of the axes of the drums. Consequently it was possible by simply shifting the platform and thereby the recording apparatus to secure from 4 to 6 complete *turns* of a *continuous* series of groups, which, with a distance of 15 to 20 feet between the tables, amounted to about

100 feet of groups and rests, requiring from thirty minutes to as much as five hours to secure, depending upon the speed of the drums.

Observers were given no other instructions than those given above. In other respects, and in so far as it was compatible with beating groups, they were left to their own devices, free to perform such other activities as studying their lessons, talking, singing, reading, or thinking, and free to have whatever ideas occurred to or in them, of whose nature no record was made or desired.

I. THE GENERAL SERIES: a–G, b–G, c–G, ... w–G, x–G, y–G, z–G

1. Groups as stable activities. In order to show that groups are stable or recurrent activities it is only necessary to show that the coefficients *a, b, c, d,* etc. are identical, or, that some of them are, and in numbers sufficient to exclude chance identities. In its ideal, i.e. extreme form, the above general series would reduce itself to one of the following forms:

$$\begin{cases} (6) & \text{a–G, a–G, a–G, a–G, a–G, a–G, etc.} \\ (7) & \text{c–G, c–G, c–G, c–G, c–G, c–G, etc.} \\ (8) & \text{etc.} \end{cases}$$

This would imply, what rarely happens except under extremely well controlled conditions, and certainly never under the conditions under which our observers worked, that the same group was always beat.

Table 1 contains a record of 17 series of groups beat on approximately seventeen succeeding days by observer R. A glance at this table will show that, considering any given series, there are groups which recur several times within that series; and considering the 17 series, that the above groups likewise occur and recur in practically all series. These facts are more conveniently shown in table 2, which constitutes a table of the frequencies of the several different groups beat by this observer for each of the seventeen days of experimentation and which give also (lower row) the total frequencies of all groups for the total of the seventeen days.

Table II. Observer R.

Table of Group Frequencies.

	5	6	7	8	10	12	13	14	15	16	17	18	19	20	21	22	23	24	26	28	29	31	32	36
1			1		1											1		1					1	
2						1				1	1	1			1			1	1					2
3					2								1			1	1			1				
4				1												3		1				1	1	
5			3	1								1	1			4								
6	4	1	5	1										1		2								
7																3								
8																2								
9																6								
10									2						1	7					1	2		
11				2												9								
12																								
13																5	1	1						
14																								
15																2							1	
16	8				2	2	1	1						1		2								
17																								
T.	12	1	9	5	5	3	1	1	2	1	1	2	2	2	2	47	2	4	1	1	1	3	3	2

	42	43	44	45	46	47	48	49	50	52	54	60	66	68	70	71	74	75	76	77	79
1			2	1	1	2	1			1				1							
2				1				1		1	1										
3	1		3	1	3									1	1						
4				1		1															1
5					5									1					1	1	
6		1		2	5		1		1												
7			1		6		2	1	1					2		1					
8					10										2	2		1			
9					2										4						
10					2	2						1			1			1			
11				1	7	2	1														
12					2	1	1								1						
13				1	6							1	1		1						
14					3	1	1							1	1						
15			1		4		1	1							1						
16			1		3												1				
17					3	2									1						1
T.	1	1	8	8	62	11	8	3	2	2	1	2	1	6	13	3	1	2	[illegible]	1	2

	93	94	95	96	99	100	104	106	108	113	117	118	130	133	136	142	143	148	156	174	190	333	393	416
1							1																	
2				1						1			2							1				
3																	1				1			
4	1				1			1		1									1					
5																			1				1	
6																								
7						1																		
8			1						1	1														
9											1											1		
10																								
11		2		1																				
12		4										1			1					1				
13			1													1								
14		3	1																					1
15		5																						
16																								
17		3	1									1		1										
T.	1	18	4	2	1	1	1	1	1	3	1	2	2	1	1	1	1	1	2	2	1	1	1	1

Considering the latter first, it is evident that there are several groups which are beat with frequencies above the average frequency of all other groups, because:

(a)	the total number of groups beat	= 303
(b)	the total number of different groups beat	= 69
(c)	the average frequency	= 4.4

The numbers 47, 62, 13, and 18 which represent the frequencies of the four groups 22, 46, 70, and 94 give the following percentage frequencies: 15.5 per cent, 22.1 per cent, 4.2 per cent and 6 per cent. These percentages are, with one exception, quite above the average frequency of all groups, and still greater than 2.4 per cent which represents the average frequency of all groups, the above four excepted. Clearly there are certain groups that are beat more often than others and more often than they would be beat, were their distribution a matter of chance. Graphically this is shown on graph 1 from which it appears that the groups 3, 7, 22, 46, 70 and 94 have maxima that are perceptibly greater than the average for all groups.

It will have to be asked and answered how often a group must be beat or must recur in any given series before it can be said that its frequency of recurrence is significant. Clearly, but only generally, this is the case when its frequency rises appreciably over the average frequency of all other groups. In this it is assumed that on a chance basis all groups are equally likely to occur. If certain groups occur more frequently and particularly when these same groups recur on different days of experimentation, or indeed in different observers, then such recurrence must have some significance. On this principle the groups 22, 46, 70 and 94 are significant, whatever the nature of the significance, and speaking generally again, the 303 groups of observer R may presently be divided into the two classes, those that are significant and those that are insignificant by the frequency criterion. Obviously no hard and fast line can be drawn between the members of the two classes of groups, and consequently the frequency principle cannot be considered as the sole or as an abso-

lute measure. Moreover this criterion would exclude from consideration those groups whose frequencies fall below that of the average of all groups and which include groups that are very significant on the bases of other criteria. For instance the group 416 occurs but a single time; yet it *can* be predicted that if the observer beats a group in the 400's, it ought to be precisely the 416 and no other 400 group. The frequency of this group is given by the fraction 1/303.

As a matter of fact the terms *significant* and *insignificant* have only a present convenience and no real justification at all except relatively speaking. Organisms do not behave on the chance basis; all their activities are significant, only some are for present purposes more so than others. There are many factors which tend to maximize the difficulties of a rigorous application of the frequency criterion, of which but one can properly be discussed at this time. Assuming that the group 416 (vide table 2) is significant for present purposes and of the same order of significance as the 22 group or the 46 group of the same observer, it might be asked how it happens that the latter groups are beat so frequently and the former only once? The answer is simple enough. The frequency criterion can only apply where or when all groups have an equal chance of recurrence, which is not in accord with either fact or theory in the present instance. In the first place, the duration of the 416 group is relatively much greater than the durations of the groups 22 and 46. In the second, the 416 is a much longer group. Theoretically it ought to be expected that animals or humans that have a relatively large repertoire of groups, some short, others of medium size, and still others that are long, will beat the medium sized groups most frequently. This is of course on the assumption that the organism in question has received no special training. A long group, or one whose duration is long, has not the same chance of recurrence, just because it takes longer for its performance, and consequently the same measure does not apply for these groups as apply for the shorter groups, and particularly for the medium sized groups.

In the light of the above the general series,

(5) a–G, b–G, c–G, d–G, e–G,............w–G, x–G, y–G, z–G

can be transformed into the series,

(6) a–G, b–G, a–G, b–G, a–G,............w–G, x–G, y–G, z–G

in which the coefficients a, b, etc., represent groups of the order of the 22, 46, 70, and 94 of observer R, and the coefficients w, x, y, and z represent groups that are for the present insignificant on the basis of the frequency criterion. The stability of the groups 22, 46, 70, and 94 is established by virtue of the possibility of the above transformation.

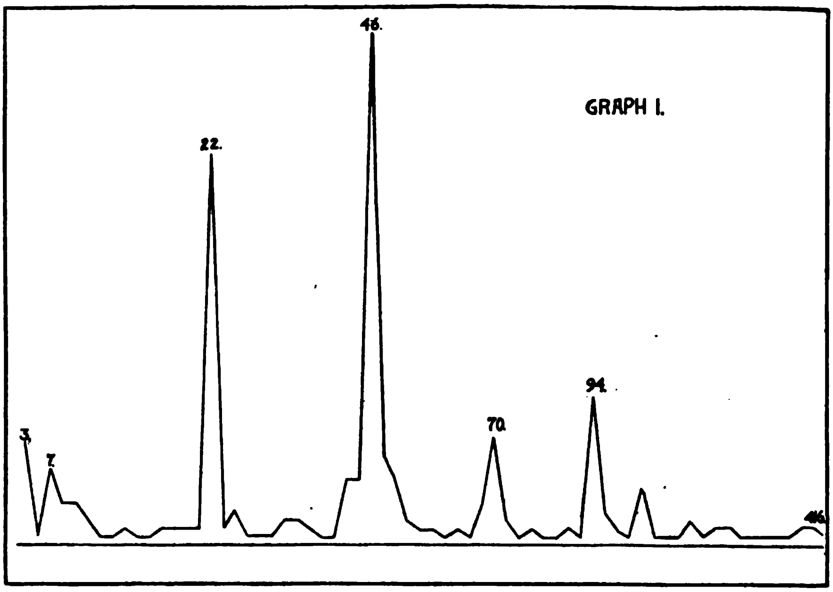
2. The coefficients a, b, c, etc. The coefficients a, b, c, d, etc., of general series (5) have been shown to be recurrent or stable in the previous section. Substituting for a, b, etc., in (6) the particular values as found for observer R by the frequency criterion,

$$a = 22; \qquad b = 46; \qquad c = 70; \qquad d = 94.$$

The facts are these: a certain individual R beats 17 series of groups on seventeen consecutive days, and of the 303 groups beat, amongst which there are 69 different groups, he beats four groups, the 22, 46, 70, and 94 relatively very frequently. The question to be asked is this; have the abnormally high frequencies of the four groups 22, 46, 70, and 94 four causes or less than four causes?

A glance at graph 1 will reveal a certain regularity or even periodicity in the maxima for the groups 22, 46, 70, and 94. It is true that the maxima for these several groups do not have even approximately the same values, which is explicable by the discussion of a previous section and also by certain facts about to be discussed. To the apparent periodicity of the above four maxima, the following other singularities may be added: considered day by day (table 2) these four groups do not have their maxima on the same days. Consider the 22 group; for whatever reason, observer R beats the 22 group with increasing frequency

as the experiment progresses up to and including the eleventh day. On the twelfth day he does not beat the 22 group at all, although he beats it again on the thirteenth day and on succeeding days. Apparently the factor which makes for the production of the 22 group, and with progressively increasing frequency, ceases for some reason to be operative on the twelfth day. The factors that make for the production of a group in general are either environmental or internal, but the former cannot enter into the discussion because the general conditions



under which the observer worked were practically identical throughout the seventeen days of experimentation. Moreover the observer's general health remained the same, and indeed there is no way in which the observer's acts on the twelfth day are explicable except on the assumption that some very radical change had taken place within his organization, which is contrary to fact. Consequently it must be assumed that observer *did* beat the 22 group, appearances to the contrary notwithstanding, on the twelfth. day. On this day observer beat

the 94 group four times, one of the maxima for this group which is not beat on the thirteenth day. Similarly, the maxima for the 70 group occurs one day later, i.e., on a different day from that of the 46 group on which day the frequency of the latter group approaches a minimum. On the tenth day a maximum of 7 obtains for the 22 group while the 70 group is beat but once and the 46 group twice. On the fifteenth when the 94 group has a maximum, all other groups practically have minimums.

Apparently an inverse correlation of some kind obtains between the frequency maxima of these four groups, and in this case some kind of a relationship must exist among the four groups. Consider finally the four *numbers* 22, 46, 70, and 94: the following numerical relations obtain:

$$\begin{aligned} 22 &= 22 \\ 46 &= 22 + 22 + 2 \\ 70 &= 46 + 22 + 2 \\ 94 &= 70 + 22 + 2 \end{aligned}$$

These relations are the counterpart of the apparent periodicities previously referred to on the graph. If the above numerical relations correspond to actual facts it would mean that

$$\begin{aligned} 22\text{–G} &= 22\text{–G} \\ 46\text{–G} &= 22\text{–G} + 22\text{–G} + 2\text{–G} \\ 70\text{–G} &= 46\text{–G} + 22\text{–G} + 2\text{–G} \\ 94\text{–G} &= 70\text{–G} + 22\text{–G} + 2\text{–G} \end{aligned}$$

and it would explain the periodicity of the maxima that obtain on graph 1 for the frequencies of the above four groups as well as the otherwise inexplicable fact that an organism can beat a certain group with progressively increasing frequency, or perform an activity of any kind in the same manner and suddenly for no apparent reason cease to perform it at all. The explanation is that the observer does not cease to beat the 22 group—he merely beats it several times in succession, hiding thereby its identity as such in the guise of an apparently new or different group. The above equations explain also why the frequencies of such groups as the 70 and 94 is relatively so small. They do not represent frequencies of different groups but rather the fre-

quency of the 22 group, and since the 70 and 94 groups each contain several 22 groups, they must recur less frequently, other things being equal.

Fortunately the proofs for the above equations were found in the records of the observer, but for reasons to be explained, only by chance. Before the records in question (containing the proofs) were made, during the last four days of experimentation, the facts as stated above were well known to the writer, who searched in vain among those records that had been made up to this time for some objective evidence which might support the conclusions to which considerations of frequency, inverse correlations, and numerical relations pointed. At present, after three years' experience the writer prefers not to refer to records for proof except under very particular circumstances because he knows that a record cannot normally contain the facts of analysis unless accidentally or incidentally,—not because the facts of numerical analyses do not correspond to behavioral actualities, but because they do not *necessarily* correspond as would be expected by ordinary habits of thought. The writer will call attention to the facts and reasons of the above at its proper time.

Consider figure 5 (first part) a fac-simile of one of the many 46 groups.

Figure 5 shows 46 beats of varying amplitudes given in a practically constant tempo. Numerical and other reasons point to the conclusion that the 46 group above consists of two 22 groups and a 2 group. Examination of the figure will show that there exist in this figure absolutely no evidence for the assumption. It cannot, except very arbitrarily be said, that here (on say the 22d beat) the first 22 group ends, and there (on say the 23d beat) the second 22 group begins, and that the last two beats constitute the above 2 group. Not only is it impossible to see in this figure the desired relationship, but any other relationships are equally invisible. The above 46 group corresponds to the generality of the 46 groups which observer R made and yet it can otherwise be shown beyond all possibility of doubt that the relation,

$$46 = 22 + 22 + 2$$

must correspond in some way and somehow to actualities of function. Consider figure 1:

A close examination of figure 1, reading it from right to left, i.e., in the order in which it was made, will reveal 21 beats of varying amplitudes followed by the 22d beat whose amplitude greatly exceeds those of its immediate neighbors. These first 22 beats may be said to constitute a 22 group. The 23d and 24th beats are relatively small and are followed by 22 beats of greater amplitude, the 22d, being again clearly larger than the 21 preceding beats. Clearly a 2 group and two 22 groups are outlined in the 46 beats which together constitute the 46 group. The

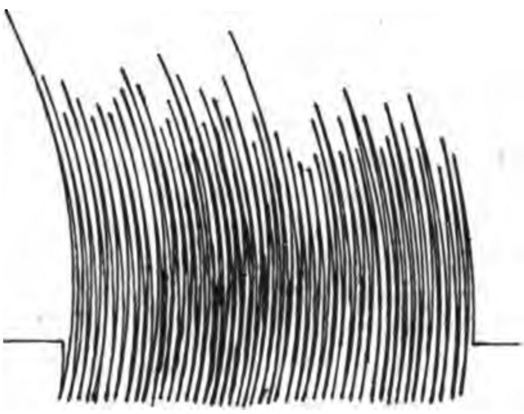


FIG. 1

technique of this delineation is by the method of "amplitude variation," but no particular significance would have attached to such variations did they not so closely correspond to prediction, or had they occurred but once. Both the latter facts indicate that the variations are not accidental, and this being so, there must exist some logical grounds for their occurrence, which cannot be other than the grounds contained in the prediction.

In his work Swindle noted and commented on the fact that the final beat of a group was often accented, that is, larger than the others. He referred to the fact by the term *final accent.* He sought to identify the accentuation with the occurrence of the

between any four beats and between every four beats measure 3 mm. Between four elements or beats, there can be inserted 2 beats and three of the smaller intervals that normally occur between any two successive elements of a group. Consequently the above intervals is *precisely* long enough to permit the interpolation of 2 beats given in the *same* tempo as that of the other beats of this group; the interpolation made in the above figure corresponds to a 46 group and is in accord with all preceding figures as well as with the prediction made with respect to the composition of the 46 group. Consider figure 4.

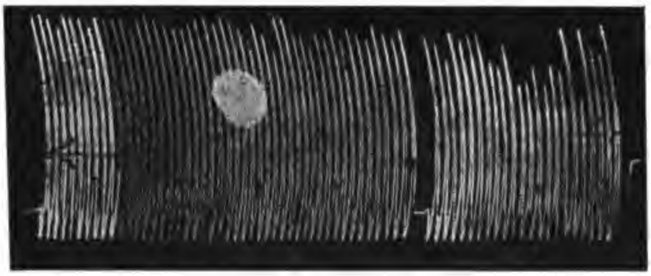


FIG. 4

Figure 4 is similar to figure 3 with the exception that a 46 group and a 22 group are concerned plus a small interval with precisely the same implications as above; two beats and only two can be interpolated, thus making a total of 70 beats which agrees with the predictions made in regard to the composition of the 70 group. Consider figure 5.

Figure 5 is an example of the same kind as the preceding. The small interval permits the interpolation of a 2 group given in the tempo of the other beats but there is a significant difference between the intervals, which significance attaches also to all the other intervals of the same kind already mentioned and to be given. Relatively the intervals are all equivalent in that they permit the interpolation of exactly 2 beats, yet absolutely the intervals are not equivalent because they have different durations.

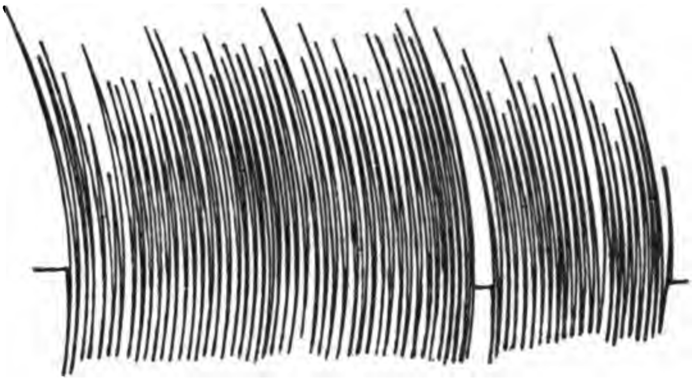


FIG. 5

Figure 6 agrees with the preceding figures except respecting the particular groups concerned. The groups are the 22 and 71 (70 + 1) plus the usual interval for the interpolation of a 2 group. It is therefore an attempted synthesis of a 95 group (94 + 1).

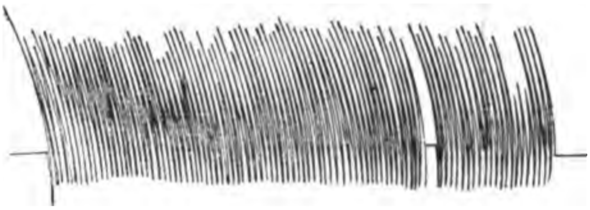


FIG. 6

Figure 7 corresponds to two groups 46 and 71 (70 + 1) plus an interval into which but one beat may be interpolated. Consequently it represents an attempted synthesis of a 118 group, a group which is beat *as such* twice by this observer.

The above are examples selected from 15 occurrences of the same kind in the work of observer R. In a certain sense they must be regarded as quite accidental because they occur relatively infrequently and consequently they cannot always be relied upon, except under pre-arranged conditions, to furnish objective evidence and proof for conditions that might be indi-

Fig. 7

cated by other circumstances. However since they have been found, they do in the present instance furnish the necessary proof for the conclusions that analysis has pointed out, and warrant the assumption that the 46, 70, and 94 groups are compounds of a single group, the 22, and that the several maxima of these groups are therefore conditioned by a common factor and not by four separate factors. The general series (6) thereby suffers the following further modification:

(7) a–G, $(2a+k)$–G, $(3a+2k)$–G, $(na+(n-1)k)$–G . . . w–G, x–G, y–G, z–G.

3. The coefficient w. A glance at table 2 and graph 1 will reveal the interesting fact that there occur in the vicinity of the maxima for the groups 22, 46, 70, and 94, groups that are beat relatively more frequently than the average of all other groups, but yet not nearly with the abnormally high frequencies of the above four groups. Their occurrence as such is not significant; their occurrence with the said frequencies is not significant; but their occurrence with their particular frequencies in *association* with the groups 22, 46, 70, and 94 is significant and raises the question whether the high frequencies of the associated groups is in any manner conditioned by the frequencies of the groups to which they are associated?

The groups to which reference is made here are the 24, 44, 45, 47, 48, 68, 71, and 95. Considering only the 46 group and its *associated* groups 44, 45, 47, and 48, the facts are as follows: the group 46 occurs with abnormal frequency; the four neighboring (in the numerical sense) groups 44, 45, 47, and 48 occur in far greater numbers than the average of all other groups, but yet with not nearly the exaggerated frequencies of the group 46. Each of the four groups 22, 46, 70, and 94 have one or more such exceptional neighbors, and with the exception of the groups 3 and 7 of which special mention will presently be made, no other group has a frequency that lies above the average. The question that logically follows is this: why are groups like the 44, 45, 47, and 48, that have relatively high frequencies associated with groups like the 46 group that has an abnormally high frequency?

When it was shown that the groups 22, 46, 70, and 94 were stable activities by virtue of their recurrence, it was assumed that this implied that the said groups were functional units, or behavioral entities of some kind, such units as reflexes, etc. Now it is well known of these more traditional units that they vary within very wide limits from time to time in their temporal and spatial attributes and in the number and sequence of their elements, as well as in their amplitudes and directions. Such variability is to be expected. Groups like the 46 above do indeed vary. Of the 62 different 46 groups no two are exactly alike except in the number of their elements from which they all derive their common name. In their other attributes they vary very widely. Particularly do they vary with respect to the amplitudes of their individual beats. The latter are obviously minor variations which, consequently, ought to occur, as they do, very frequently on grounds of probability. But variations of 1, 2, 3, or more beats cannot be said to be minor variations and therefore would not occur as often as amplitude variations, not necessarily because they are essentially different kinds of variations but because they imply greater variations of the same kind. Thus a variation of one beat from the typical 46 group implies theoretically only that the 46th beat has an amplitude 0 which is a greater deviation from the 46 group than 45 beats with a normal amplitude plus one beat with an amplitude of about one-third of the average of the other beats would be.

Since amplitude variations do occur very frequently and since variations of one beat or more are of the same nature as these, differing in degree only, and since they therefore imply relatively large variations of amplitude, it is to be expected on purely a priori grounds that a typical group, as the 46 group, will have associated with it groups like itself and of the same species whose frequencies will in general be the greater, the greater the frequency of the type group; and the less, the farther the individual members of the same species are numerically removed from the type group. This is obviously the case in the present instance where the above conditions are sufficiently fulfilled as on graph 1.

Accordingly groups like the 44, 45, 47, and 48 will be considered as varieties of the group species 46, and it will be predicted that in any similar investigation such varieties will occur. The coefficient w thereby suffers the following transformations:

$$w_1 = a \pm n, \quad w_2 = (2a + k) \pm n; \quad w_3 = (3a + 2k) \pm n;$$
$$w_4 = (na + (n-1)k) \pm n.$$

4. The coefficient x. The general series (6) contains the undefined coefficient x whose nature it is the purpose of this section to discover. The simple facts upon which the discussion is based are as follows: an organism beats a certain group a very often; it was also shown that he beats groups of the order $2a$, $3a$, $4a$, etc. The question to be asked is whether any other multiples of a might be expected to occur which cannot be demonstrated to be such by an appeal to simple frequency criteria?

The following numerical equations obtain between those groups that have been shown to be multiples of the group 22:

$$\begin{aligned} 22 &= 22 \\ 46 &= 22 + 22 + 2 \\ 70 &= 46 + 22 + 2 \\ 94 &= 70 + 22 + 2 \end{aligned}$$

Continuing these equations with their numerical implications, the following expressions can be obtained:

$$\begin{aligned} 118 &= 94 + 22 + 2 & \qquad 262 &= 238 + 22 + 2 \\ 142 &= 118 + 22 + 2 & 286 &= 262 + 22 + 2 \\ 166 &= 142 + 22 + 2 & 310 &= 286 + 22 + 2 \\ 190 &= 166 + 22 + 2 & 334 &= 310 + 22 + 2 \\ 214 &= 190 + 22 + 2 & 358 &= 334 + 22 + 2 \\ 238 &= 214 + 22 + 2 & 382 &= 358 + 22 + 2 \\ & & 416 &= 382 + 22 + 2 \end{aligned}$$

In the above equations the numbers on the left hand side represent groups that might be expected to be beat by this observer in view of the fact that he beats the 46, 70, and 94 groups. Of these, the following groups are beat with the indicated frequency by this observer:

118	2 times,	333 (334–1)	1 time,
142	1 time,	416	1 time,
190	1 time,		

It is hardly necessary to comment on the remarkable accuracy of the above predictions and it is believed that the concurrence that obtains between prediction and fact is sufficiently great to warrant the belief that the groups 118, 142, 190, 333, and 416 are of the same order as the 46, 70, and 94, and therefore of the nature of the 22 group. It should be pointed out however that frequency criteria cannot be invoked in the cases of these groups to establish the said relations partly because they are very large groups and partly also for reasons later to be considered. Since the general series suffers no radical modification by the above discussion, no transformations will be made; the general coefficient x may be taken to stand for all multiples of the type group a that can be predicted as in the above.

5. The coefficient a. With the exception of the general coefficients y and z, all other coefficients have been shown to be definable in terms of the coefficient a. It is interesting to examine whether a can be expressed in terms other than itself. A rigorous application of the mathematical law of *no exception* would lead to the conclusion that such groups as the 22 might be analyzed and expressed in terms of groups smaller than itself in much the same manner as the groups 46, 70, and 94 have been expressed in terms of the 22 group.

The application of the above principle however carries with it the necessity for giving to its results a very clear meaning which may not be simply conventional or arbitrary, but which must have some relation to, and which will square with, the morphology and physiology of the organism which has produced the groups in question. In consequence thereof analyses may be carried out only so far as they are possible of interpretation, the limits and meanings of which we are about to examine.

The facts or data on which an analysis of the 22 group might be possible are these: the hypothetical components must necessarily be smaller than the 22 group; of the smaller groups which

observer R beat, there must be some definite reason or the selection of a given set of groups as the components, i.e., he selection may not be arbitrary; with a possible exception there are no objective proofs to guide in the selection of these components; the frequency criterion is the sole remaining guide, which points out the groups 3, 7, 8, and 10 as componential possibilities of the 22 group because of their recurrence.

The groups 3, 7, 8, and 10 have relatively high frequencies. This cannot be due to chance and inasmuch as all other groups whose frequencies are above the average have been shown to be related to the 22 group there can be little question that the latter

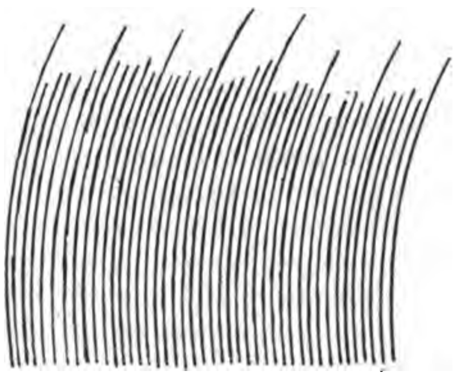


FIG. 8

is related in some way to one or more of the groups 3, 7, 8, and 10. A bit of objective evidence which may be offered in support of the conclusion is the fact that the 416 group which has been shown to be a multiple of the 22 group, is made up of fifty-seven 6 groups and two 7 groups. A portion of the 416 group is shown in fac-simile in figure 8.

Whether or not the coefficient *a* or as in the present example, the 22 group, can be reduced to lower terms is in a certain sense not important. The most important fact about the 22 group is that it is obviously a functional unit and a functional entity of a very definite kind in terms of which many other groups may be expressed. If reducible, the same importance would attach

to the then resulting group as now attaches to the 22 group, and since for present purposes analysis must cease at some point, it might as well be at the 22 group as elsewhere. The importance that attaches to the 22 group, as it stands at present for the reasons cited, does not depend upon the fact whether the 22 group is or is not an ultimate functional unit, if such exist; it depends solely on the fact that the 22 group is a functionial entity and that it is related to other groups in very definite ways. For that reason the question as to its composition will be left open for the present with this additional remark, that it is without doubt, or *was* at some time a complex activity, compounded of smaller units whose precise nature is at this time indeterminable.

6: The coefficient y. The general series (6) contains a coefficient y which represents a class of groups whose description and analysis have not been possible thus far. It is the purpose of this section to consider this class of groups and to this end the data of observer S will be studied (table 3).

The question must first be raised and answered on what assumptions it is possible to attempt a description and definition of a general series of groups such as series (6) by means of an appeal to the data of *several* different observers, defining certain of the coefficients of this series on the basis of the groups of one observer and others on that of different observers. The question contains the implication that different observers beat such radically different groups or classes of groups that no generalizations are possible, or otherwise it ought to be possible to completely define *every* coefficient of the general series (6) by the data of a single observer, such as R.

It is not the same to say that different observers beat different groups and different observers beat different classes of groups. Different observers do beat different groups, though they often also beat similar or even identical groups. On probability grounds this is to be expected considering the large number of possible observers. But the statement that observers beat different classes of groups contains the contrary to fact implication that members of the same species differ very widely in structure and function, i.e., that they differ more than they are alike.

Since members of the same species are more alike than they are different, it must be true, or expected, that the laws governing their habits of behavior, and the combination of these, and their succession will generally be the same, particularly when good reasons can be assigned for *apparent* deviations from the above assumption.

TABLE 3

Observer S

1.) 24, 33, 57, 73, 76, 45, 11, 55, 52, 38, 24, 35, 40, 5, 38, —, 21, 113, 11, 48, 78, 23, 79, 198, 53, 77, 66, 20, 131, 22, 37, 10, 143, 80, 20, 11, 36, 32, 38.
2.) 33, 27, 74, 87, 83, 45, 128, 23, 258, 27, 256, 39, 54, 240, 15, 176, 124, 58, 22, 154, 57, 20, 248.
3.) 33, 162, 32, 56, 180, 53, 55, 282, 105, 55, 238, 41, 220, 207, 37, 100, 95, 30, 31, 97, 338, 146, 144, 337, 199, 18, 47, 264, 73, 126, 237, 32, 176.
4.) 44, 51, 480, 380, 170, 153, 98, 207, 435, 120, 176, 209, 404, 45, 84, 81, 85, 39, 80, 105, 58, 32, 141, 120, 110, 18, 141, 174, 17, 161, 103, 982.
5.) 64, 112, 62, 173, 75, 83, 279, 118, 65, 148, 144, 47, 144, 144, 192, 142, 38, 163, 134, 59, 137, 156, 115, 27, 93, 277, 90, 132, 111.
6.) 57, 103, 89, 29, 125, 145, 246, 65, 256, 61, 197, 96, 163, 368, 176, 212.
7.) 33, 129, 50, 84, 80, 89, 180, 94, —, 68, 137, 193, 57, 156, 96, 80, 59, 167, 60, 194, 190, 124, 34, 110, 43.
8.) 65, 100, 35, 116, 162, 46, 162, 120, 171, 90, 38, 63, 233, 42, 75, 35, 147, 92, 94, 102, 127, 151, 156, 47, 128, 97, 235, 34, 45.
9.) 28, 69, 90, 131, 41, 107, 255, 47, 96, 87, 183, 60, 169, 49, 141, 156, 188, 94, 299, 167, 57, 225, 52, 95.
10.) 92, 101, 113, 57, 196, 190, 47, 337, 57, 254, 103, 77, 277, 60, 245, 15, 41, 60, 240, 41, 67, 46, 99.

For purposes of convenience the second record of observer S is reproduced below in the order in which it was made or beat.

33, 27, 74, 87, 83, 45, 128, 23, 258, 27, 256, 39, 54, 240, 15, 176, 124, 58, 22, 154, 57, 20, 248

The following facts are of interest in connection with the work of observer R. Observer S beats the 27 group twice and its double, the 54 once; the 128 group and its double, the 256 group, and also the group 258 which differs from the double of the 128 by two beats, making it very likely a variety of the 128 or 256 group species; the 22 group and the multiples 176 and 154; the 124 group and its double, the 248 group. In this respect then, the present observer does not differ from ob-

server R at all, except that he does not seem to beat groups of type *a* and their *multiples* as frequently as R. This difference however is very significant as will appear hereafter, and it is precisely such differences which make generalizations or group classifications on the basis of a single observer difficult.

Consider the first four groups of the above series of groups, namely:

33, 27, 74, 87,

and the corresponding *numerical* equation:

$$33 + 27 + 27 = 87. \tag{1}$$

and the next four groups of this series,

83, 45, 128, 23,

and the corresponding *numerical* equation:

$$83 + 45 = 128. \tag{2}$$

and the following seven groups:

258, 27, 256, 39, 54, 240, 15,

and the corresponding *numerical* equations:

$$\begin{array}{lll}
(3) & (9 \times 27) + 15 & = 258 \\
(4) & 39 + 54 + 39 + 54 + 54 & = 240 \\
(5) & 27 + 27 & = 54 \\
(6) & 39 + 15 & = 54 \\
(7) & (16 \times 15) & = 240
\end{array}$$

and finally the series:

176, 124, 58, 22, 154, 57, 20, 248,

and the corresponding *numerical* equations:

$$\begin{array}{lll}
(8) & (8 \times 22) & = 176 \\
(9) & 58 + 22 + 22 + 22 & = 124 \\
(10) & 57 + 20 + 57 + 20 & = 154 \\
(11) & (7 \times 22) & = 154 \\
(12) & 124 + 124 & = 248 \\
(13) & 57 + 57 + 57 + 57 + 20 & = 248
\end{array}$$

The question to be answered is this: what relations exist between any of the above short series of groups and the corresponding *numerical* equations? It must be noticed first that all the numbers which occur in equations (1) to (13) occur also in that part of the series to which they correspond, and that no other number which does not occur in the corresponding part of the series occurs in the equations. Two possible conclusions can be drawn from these correspondences; either we are concerned with a remarkable set of coincidences or the above numerical relations have some significance. The first conclusion is untenable from the fact that such numerical relations as between the coefficients of groups are by far too common to be regarded as accidents, and secondly from the fact that in principle the above equations do not differ from similar equations obtained in the case of observer R except that in the latter case the equations involved the addition of *identical* coefficients whereas here they involve the addition of *non-identical* coefficients. From this it is obvious that the general coefficient y may be taken to represent groups whose coefficients are obtainable by the addition of two or more *different* groups which latter groups must occur in the temporal vicinity of the groups whose components they may be said to be.

An extension of the arithmetical principles involved in the above equations is possible when they are obtained not only as between neighboring groups of the same record, but also between any groups at all within the said record. Below are given in categorical form a list of the more important equations obtainable from among the several groups of the series under discussion. With certain exceptions, the equations *do* not contain any numbers that are *not* found as such among the coefficients of the groups of the above series. The exception mentioned above consists of the use of the *half* of the coefficient of groups in the case of even numbered groups, and approximate *halves* or so-called *physiological halves* of odd numbered groups. The only justification for the use of said numbers in the making of analytical equations as above comes from the fact that it is so generally possible to do so, and from the obvious relation that

exists as between halves and the multiples of a number. Thus we may consider the 46 group the *physiological double* of the 22 group, or we may consider the 22 group the *physiological half* of the 46 group.

A partial list of such equations follows:

$$
\begin{aligned}
74 &= 33 + 27 + 27/2\ (14) \\
&= 27 + 27 + 20 \\
&= 54 + 20 \\
&= 39 + 15 + 20
\end{aligned}
$$

$$
\begin{aligned}
87 &= 33 + 27 + 27 \\
&= 33 + 54 \\
&= 33 + 39 + 15 \\
&= 57 + 15 + 15 \\
&= 27 + 45 + 15 \\
&= 27 + 15 + 15 + 15 + 15
\end{aligned}
$$

$$
\begin{aligned}
83 &= 33 + 27 + 23 \\
&= 33 + 33 + 33/2 \\
&= 23 + 45 + 15 \\
&= 23 + 23 + 22 + 15 \\
&= 23 + 15 + 15 + 15 + 15
\end{aligned}
$$

$$
\begin{aligned}
45 &= 15 + 15 + 15 \\
&= 23 + 22
\end{aligned}
$$

$$
\begin{aligned}
128 &= 83 + 45 \\
&= 74 + 54
\end{aligned}
$$

and all substitution products.

$$23 = 23$$

$$
\begin{aligned}
258 &= (128 + 1) + (128 + 1) \\
&= 83 + 23 + 23 + 83 + 23 + 23 \\
&= 45 + 45 + 45 + 45 + 45 + 33 \\
&= 33 + 33 + 33 + 33 + 33 + 33 + 33 + 27 \\
&= 27 + 27 + 27 + 27 + 27 + 27 + 27 + 27 + 27 + 15
\end{aligned}
$$

and all substitution products.

$$27 = 27$$

$$\begin{aligned} 256 &= 83 + 45 + 83 + 45 \\ &= 128 + 128 \\ &= 54 + 74 + 54 + 74 \end{aligned}$$

and all substitution products.

$$39 = 39$$

$$\begin{aligned} 54 &= 27 + 27 \\ &= 39 + 15 \end{aligned}$$

$$\begin{aligned} 240 &= 39 + 54 + 39 + 54 + 54 \\ &= 33 + 27 + 33 + 27 + 33 + 27 + 33 + 27 \\ &= 176 + 27 + 15 + 22 \\ &= 124 + 83 + 33 \\ &= 124 + 39 + 23 \\ &= 124 + 74 + 27 + 15 \\ &= 128 + 58 + 54 \\ &= 128 + 27 + 45 + 20 + 20 \\ &= 83 + 83 + 74 \\ &= 74 + 87 + 57 + 22 \\ &= 74 + 87 + 39 + 20 + 20 \\ &= 45 + 45 + 45 + 45 + 45 \end{aligned}$$

and all substitution products.

$$15 = 15$$

$$\begin{aligned} 176 &= 22 + 22 + 22 + 22 + 22 + 22 + 22 + 22 \\ &= 154 + 22 \\ &= 124 + 15 + 15 + 22 \\ &= 57 + 22 + 45 + 15 + 22 + 15 \\ &= 58 + 58 + 15 + 15 + 15 + 15 \\ &= 128 + 33 + 15 \\ &= 33 + 33 + 33 + 33 + 22 + 22 \\ &= 33 + 33 + 33 + 57 + 20 \\ &= 33 + 33 + 45 + 45 + 20 \\ &= 27 + 27 + 27 + 27 + 27 + 27 + 27/2 \\ &= 27 + 27 + 27 + 27 + 33 + 15 + 20 \\ &= 74 + 57 + 45 \\ &= 74 + 87 + 15 \\ &= 39 + 39 + 39 + 39 + 20 \\ &= 39 + 54 + 83 \end{aligned}$$

and all substitution products.

Below are given in a convenient form a few other examples of the same kind from the records of observer T and S. In every case the group to be analyzed is starred and below it under the horizontal line are given the several sets of components found for the group above the line:

Résumé of record 3, observer S

$$\frac{97^*}{47 + 32 + 18}$$

$$\frac{338^*}{\dfrac{97}{47 + 32 + 18} + \dfrac{144}{47 + 47 + 32 + 18} + \dfrac{97}{47 + 32 + 18}}$$

$$\frac{146^*}{73 + 73}$$

$$\frac{144^*}{47 + 47 + 32 + 18}$$

$$\frac{337^*}{\dfrac{146}{73 + 73} + 47 + \dfrac{144}{47 + 47 + 32 + 18}}$$

$$\frac{199^*}{73 + \dfrac{126}{47 + 47 + 32}}$$

$$18^*$$

$$47^*$$

$$\frac{264^*}{\dfrac{\dfrac{126}{47 + 47 + 32} + 73}{199} + 18 + 47}$$

$$\frac{73^*}{18 + 18 + 18 + 18\,(?)}$$

$$\frac{126^*}{47 + 32 + 47}$$

$$32^*$$

$$\begin{array}{c} 176^* \\ \hline 144 \quad + \quad 32 \\ \hline 47 + 47 + 32 + 18 \end{array}$$

Résumé of record 8, observer S

$$65^*$$

$$\begin{array}{c} 100^* \\ \hline 35 + 65 \end{array}$$

$$35^*$$

$$\begin{array}{c} 116^* \\ \hline 35 + 46 + 35 \end{array}$$

$$\begin{array}{c} 162^* \\ \hline 116 \quad + \quad 46 \\ \hline 35 + 46 + 35 \end{array}$$

$$46^*$$

$$120^*$$

$$90^*$$

$$38^*$$

$$63^*$$

$$\begin{array}{c} 233^* \\ \hline 90 + 38 + 63 + 42 \end{array}$$

$$42^*$$

Résumé of record 4, observer T

(To be read from left to right.)

$$12,^* \qquad \begin{array}{c} 71^* \\ \hline 12 + 59 \\ \hline 12 + 47 \end{array} \qquad \begin{array}{c} 81^* \\ \hline 22 + 59 \\ \hline 47 + 12 \end{array} \qquad 22^* \; 29^*$$

$$\begin{array}{c} 110^* \\ \hline 29 + 81 \\ \hline 22 + 59 \\ \hline 12 + 47 \end{array} \qquad 47^* \qquad \begin{array}{c} 103^* \\ \hline 59 + 22 + 22 \\ \hline 47 + 12 \end{array}$$

$$\frac{59^*}{47+12} \qquad \frac{262^*}{\frac{103}{50+22+22} + \frac{59}{47+12} + \frac{71}{12+\frac{59}{12+47}}}$$

Résumé of record 7, observer T

$$8,^* \; 14^* \; 14^* \qquad \frac{38^*}{8+14+16} \qquad 16^* \qquad \frac{174}{10 \times 16 + 14} \qquad 89^*$$

$$\frac{94^*}{10 \times 8 + 14} \qquad \frac{103^*}{6 \times 16 + 12}$$

All observers whose records have been cited above, observed while they were beating a "feeling" that they should or could cease beating at certain times, for some reason however not doing so. Observers were instructed to accent the beats at which the above feelings were perceived by tapping a little harder. These accents would accordingly divide the whole group into a number of successive groups whose limits were conditioned by, or accompanied by, the said mental states or "feelings." It was desired to ascertain whether or not these mental states had any behavioral correlates or whether the instructions to accent would as it were "salt out" groups that were known to be, or which could be shown to be, functional entities in the sense in which this term is used in the present study. In case it happened that the accents, thus determined, coincided with or conditioned groups that were entities, it would mean that the above "feelings of being able to stop" had objective correlates, and that they existed because the organism in question at other times did actually stop beating when the last element of this or that group had been made.

As will be seen from the examples to be cited, the facts agree with expectations, but certain factors make it impossible generally to make use of this technique for the purpose of salting out groups. The instructions to accent a beat or beats, when the said mental states obtained, themselves act as a stimulus for having these same mental states at times where they would not have been had, had no instructions been given. The result is that, while at first a certain modicum of success will attend

the use of the technique, presently the number of accented beats increases so that not only is it impossible to use the data obtained in this way, but it "spoils" an otherwise good observer whose habits become continually modified by the instructions given. This condition no doubt obtains generally, particularly in introspections, and it was for this reason that the writer not only did not require introspections, assuming them to have value, but hesitated to mention the word *introspection* lest the habits of his observers might become too radically changed either by actual instructions or by casual references to it.

Observer Q beat the following series of groups under conditions as above:

166 81 225 148 170

The 166 group was accented on the 83 beat and on the 137th beat. The 83d beat divides the 166 group into two equal parts, namely, two 83 groups. Thus the present observer agrees with others cited in this work. The 137th beat is the 54th beat of the second hypothetical 83 group, thereby dividing this group into the two groups 54 and 29. The 81 group following, which differs only by 2 from the two 83 components of the 166 group is accented on the 54 beat, i.e., thereby separating out another 54 group, and dividing it into the groups 54 and 27 which latter group equals 54/2. The 225 group was accented only on the 54th beat, and the rest of the series is unanalyzable.

The same observer beat the following series:

66 86 161 117 42

The group 66 was accented on the 43d beat, i.e., on the 86/2 beat, which 86 group follows the 66 group. The group 161 was accented on the 57th beat, thereby dividing it into the groups 75 and 86. The 86 groups actually preceded the 161. The 117 was accented on the 75th beat, dividing it into the groups 75 and 42. The 75 was shown to be a similar component of the preceding group, and the 42 group actually follows the 117 group.

From the above it would seem possible to conclude that the coefficient y of the general series (6) is definable in terms of

the sum of two or more coefficients of the type a_1, a_2, a_3, etc., where a_1, a_2, a_3, etc., are of the order a and definable as this one has been defined. Thus a peculiar difference obtains between the observers R and S. Where R beats but a single type group a, (22) and its multiples or physiological multiples, and beats these relatively very frequently, S beats several of the type a, as a_1, a_2, a_3, etc. but does not beat any of these with nearly the high frequencies obtainable in R. Whether or not the coefficients a_1, a_2, a_3, etc., can themselves be defined in lower terms, or in terms of a coefficient a is insignificant for present purposes. It is only significant that such coefficients exist, and that the groups they represent are functional entities in terms of which a large number of other groups can be defined. As previously stated, there can be but little question that if a present relation does not obtain between the above type groups, a genetic relationship does obtain which would mean that these groups perhaps once had a common ancestor functionally considered.

7. The coefficient z. The general coefficient z will usually represent all groups that are not capable of analysis as indicated in the previous sections of this study. It may be interesting to point out why it will not always be possible to analyze the totality of any given series of groups, and why there will be found certain individuals whose groups will be generally not capable of analysis.

The chief reason why this must be so is that the environment even in the best controlled experiments will play its customary rôle in modifying behavior of any kind and particularly the habits under present discussion. The present writer took only the most general precautions to insure a more or less constant environment for the reason that it is generally quite impossible to realize even the faintest approximation to this ideal except where observers are totally devoid of all senses and memory, which would make them worthless for our purposes, but more so because such conditions, whether realized either wholly or in part, are so unnatural and so unusual that they themselves constitute one of the gravest sources of stimulation, and constantly condition corresponding modifications of their habits,

a very undesirable goal in investigations like the present one. The habits in the possession of an organism have generally been acquired under every-day conditions and they are therefore least liable to change under the conditions under which they were developed than under any other conditions.

Since an observer cannot, in general, be entirely removed from sources of environmental stimulation, it must happen now and then, depending on the one hand on the degree of the constancy of the environment or on the "habitualness" of the environment, and on the other on the constitution of the organism itself, that groups will be beat whose nature depends essentially on the environment and not on the organization of the organism, which, if it could be accurately measured and defined, would establish the identity of *every* group of any observer.

8. Other analytical methods. It occurred to the writer to extend the methods so generally successful in the consideration of a single series of groups of any given observer to a comparative study of several records of the same observer for the purpose of ascertaining whether the same general relations obtained for the same group in different series. For this purpose, records of the same observer (beat on different days) which contained a common group were selected and compared. The comparisons were effected between the groups that immediately preceded or followed the group common to the several series.

In records 3 and 10, observer S beat the following series:

$$\begin{cases} 57 & 196 & \mathit{47} & 337 & 57 & 354 \\ 199 & 18 & \mathit{47} & 264 & 73 & 126 \end{cases}$$

The common group in the two records or parts of records is the 47. If a simple relation obtains between any pair of groups that have about the same temporal positions with respect to the common group 47, or any common group, or any groups that are known to be related, then the significance of the relations that obtain for the groups of any single series is thereby strengthened because the corresponding probability problem becomes more restricted. Another condition or another set of conditions

enter into the problem to which the coefficients of the series must conform.

Subtract the number 264 from the number 337 and a *number* 73 obtains. But this *number* is identical with the coefficient of the group 73 which follows the 264 group. From this it appears, that while observer S follows the 47 group by the 337 in record 3, he also follows this group by the 337 group in record 10 with this difference only, that in 10 he beats the 337 group in two parts, while in 3 he beats the 264 and 73 groups in strict succession.

Records 3 and 4, observer S, give the following series:

18	47	264	73	126	237
18	141	174	17	161	103

The following equations indicate some of the relations here found:

$$(1)\quad 141 = 47 + 47 + 47 \qquad (3)\quad 174 = 73 + 47 + 18 + 18 + 18$$
$$(2)\quad 264 = 174 + 73 + 17 \qquad (4)\quad 161 = 126 + 17 + 18$$

The complex may therefore be written in the convenient form:

$$\left\{\begin{array}{l} 18^* \quad 47^* \quad \underset{\underset{47\,+\,73\,+\,18\,+\,18\,+\,18}{\underline{174}}\;+\;17\,+\,73}{\underline{264^*}} \quad 73^* \quad 126^* \\[2em] 18^* \quad \dfrac{141^*}{47+47+47} \quad \dfrac{174^*}{47+73+18+18+18} \quad \dfrac{17^* \quad \dfrac{161^*}{17+128+18}}{73+17+17+18+18+18} \end{array}\right.$$

The following series are taken from the Records of Ob. T.:

8	12	*16*	12	13
25	56	*16*	72	36
12	38	*16*	174	89
89	94	*16*	108	—
15	13	*16*	25	11

Analyzed, the following relations are found:

$$8^* \qquad 12^* \qquad \frac{16^*}{8+8} \quad 12^* \qquad 13^*$$

$$\frac{25^*}{12+13} \quad \frac{56^*}{12+12+12+12+8} \quad \frac{16^*}{8+8} \quad \frac{72^*}{6 \times 12} \quad \frac{36^*}{12+12+12}$$

$$12^* \quad \frac{38^*}{12 \times 13 + 13} \quad \frac{16^*}{8+8} \quad \frac{174^*}{108 + 25 + (13 \times 3)} \quad \frac{89^*}{(5 \times 13) + (2 \times 12)}$$

$$\frac{89^*}{(5 \times 13) + (2 \times 12)} \quad \frac{94^*}{56+38} \quad \frac{16^*}{8+8} \quad \frac{108^*}{(9 \times 12)}$$

$$15^* \qquad 13^* \qquad \frac{16^*}{8+8} \quad \frac{25^*}{12+13} \qquad 11^*$$

The following series are taken from the records of S:

	33	162	32	56
	33	129	50	84
	33	27	74	87
24	*33*	57	73	76

Analyzed, they take the following form:

	33*	162* 33 + 129 57 + 24 + 24 + 24 33 + 24 96 32 + 32 + 32	32*
	33*	129* 57 + 24 + 24 + 24 33 + 24 96 (1) 32 + 32 + 32 27 + 27 + 27 + 24 + 24	$\frac{50^*}{33 + 33/2}$
	33*	27*	$\frac{74^*}{33 + 27 + 27/2}$
24*	33*	$\frac{57^*}{24 + 33}$	$\frac{73^*}{33 + 27 + 27/2}$

(1) The 96 group appears further on in the first series and therefore cannot be shown.

In records 2, 7, and 9, observer S beat the following series whose common group, the 57, does not appear for want of space:

—	33	129	50	84	80	89
48	96	86–7	183 (2) 182 + 1 134 + 48	60	169 (2) 142 + 27	49
—	258	27	256	39	54	240

(2) These groups were objectively analyzed into the indicated groups by the method of accents; without instructions.

Analyzed, they take the following form:

—	33*	129* 33 + 96 48 + 48 33/2 + 27 + 86 27 + 27 + 32 54	50* 33 + 33/2
48*	96* 48 + 48	87* 1 + 86 32 + 27 + 27 33	183* 182 + 1 48 + 134 48 + 86 96 87 48 + 134 + 1 84 + 50 60 + 84 + 39 33 + 27
—	258* 129 + 129 33 + 96 48 + 48	27*	256* 138 + 128 45 + 83 74 + 54 134 + 50 + 27 86 + 48 33 + 27 + 27